(-1) signs.

Math 2E Quiz 2 Morning - April 7th

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Show all work, and try to simplify your answers. *There is a question on the back side.

1. [10pts] Evaluate the following integral by changing the order of integration:

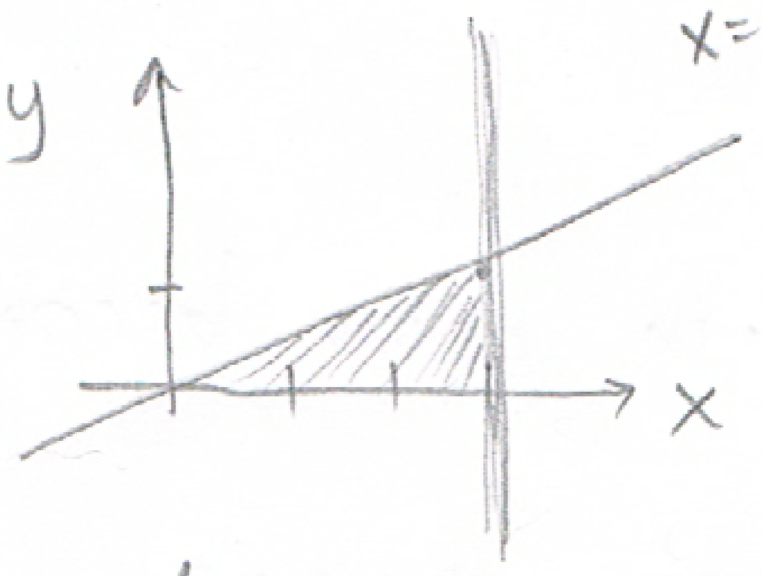
$$\int_0^1 \int_{3y}^3 e^{x^2}\, dx dy.$$

It should help to draw the domain of integration first.

$x=3$

$x=3y\ ;\ y=\frac{x}{3}$

From integral bounds, $3y \le x \le 3$ $\Rightarrow$
$0 \le y \le 1$



So, our new bounds yield, when flipping order,

$$\int_{x=0}^{x=3} \int_{y=0}^{y=\frac{x}{3}} e^{x^2}\, dy\, dx \quad +4 \quad = \int_{x=0}^{x=3} y e^{x^2} \Big|_{y=0}^{y=\frac{x}{3}} dx$$

$$= \int_0^3 \frac{x}{3} e^{x^2}\, dx \quad +1$$

$u = x^2$
$du = 2x\,dx$

$$= \int_{u=0}^{u=9} e^u \frac{du}{6}$$

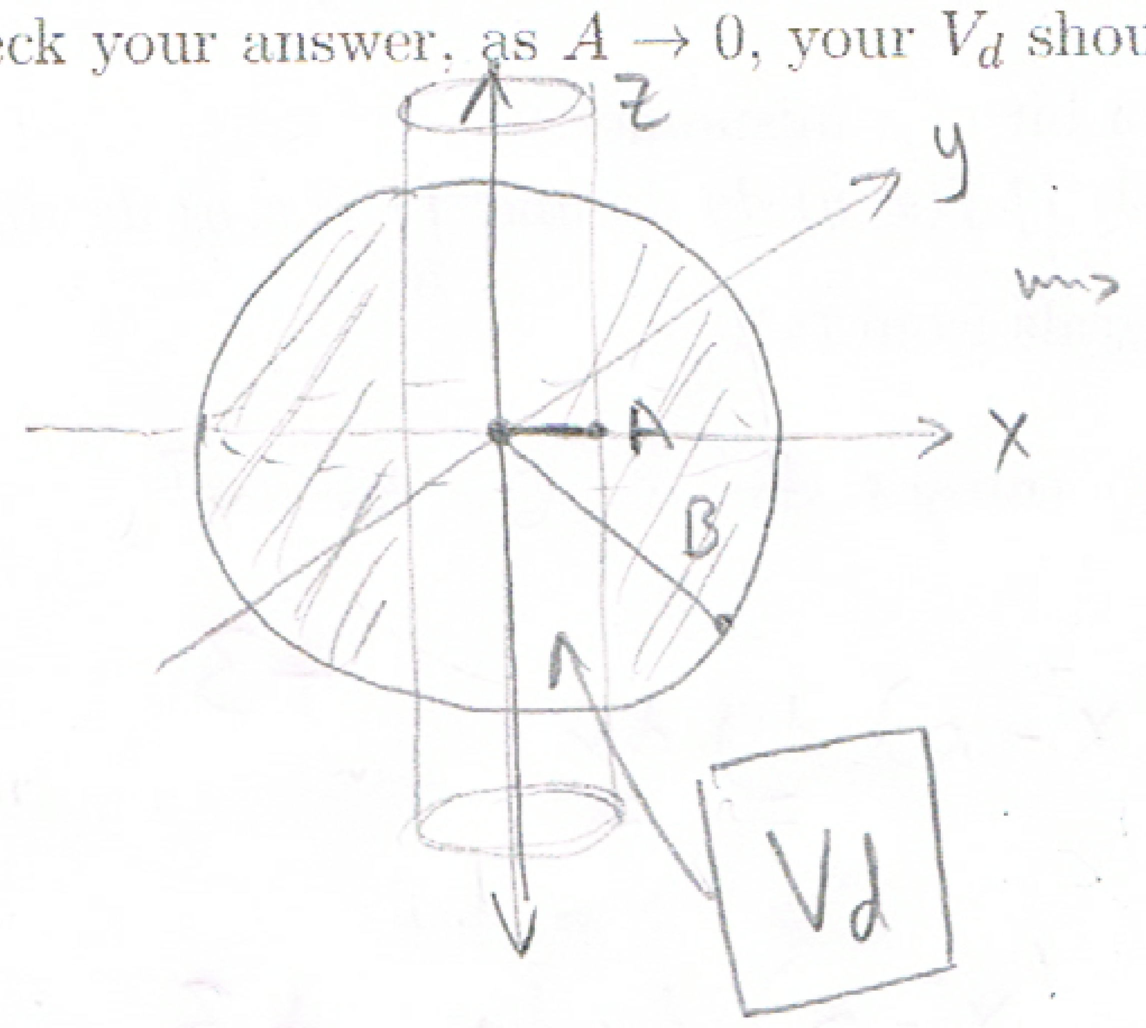
$$= \frac{1}{6} e^u \Big|_{u=0}^{9} \quad +4$$

$$= \boxed{\frac{1}{6}(e^9 - 1)} \quad +1$$

2. [10pts] Consider the problem where a cylindrical drill with radius A is used to bore a hole through the center of a sphere of radius B (where $0 < A \le B$). One approach to this homework problem is to first find the find volume that was drilled out, call it V_d, and then subtract this from the volume of the whole sphere.

Compute V_d, the volume of the sphere that was drilled out.
To check your answer, as $A \to 0$, your V_d should go to zero, too.

Picture:



⤳ Top View:

- We still integrate by polar coordinates. The drilled region has $0 \le r \le A$, $0 \le \theta \le 2\pi$.
- sphere eqn is $x^2+y^2+z^2 = B^2$, so $z = \pm\sqrt{B^2-x^2-y^2}$.

This means that our "height" to integrate is $(z_{top} - z_{bot})$, $z_{top} = \sqrt{B^2-x^2-y^2}$

$z_{bot} = -\sqrt{B^2-x^2-y^2}$

so $z_{top} - z_{bot} = 2\sqrt{B^2-x^2-y^2} \overset{\text{Polar}}{=} 2\sqrt{B^2-r^2}$

$$\hookrightarrow V_d = \int_0^{2\pi}\int_0^A 2\sqrt{B^2-r^2}\, r\,dr\,d\theta \quad ; \qquad +4$$

Let $u = B^2 - r^2$
$du = -2r\,dr$,

$$= \int_0^{2\pi}\int_{u=B^2}^{u=B^2-A^2} \cancel{2}\sqrt{u}\cdot\frac{du}{-\cancel{2}}\cdot d\theta$$

$$= \int_0^{2\pi} -\frac{2u^{3/2}}{3}\Bigg|_{B^2}^{B^2-A^2} d\theta$$

// constant in θ so we just get length of θ-interval

$$= \int_0^{2\pi} \frac{2}{3}\left(B^3 - (B^2-A^2)^{3/2}\right) d\theta \qquad +4$$

$$= \boxed{\frac{4\pi}{3}\left(B^3 - (B^2-A^2)^{3/2}\right)} \qquad +2$$

- As $A \to 0$, we get $\frac{4\pi}{3}(B^3 - B^3) = 0$ ✓